Dossier Pierre Duhem

Poincaré and Duhem: Resonances in their First Epistemological Reflections

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Abstract: The object of this article is to show a certain proximity of Duhem to Poincaré in his first philosophical reflections. I study the relationships between the scientific practices of the two scholars, the contemporary theoretical context and their reflections. The first part of the article concerns the changes in epistemological consensus at the turn of the century. The second part will be devoted to Poincaré's reflections on the status of physical geometries and physical theories, as they appear in his texts written around 1890. Then I analyze the first reflections of Pierre Duhem on physical theory, in particular his thesis of the hypothetical-symbolic character of physical theories and his criteria for selecting good theories, partly associated with his ideal of physical theory; the whole set of considerations, highlighting the Poincarean inspiration.

Keywords: Pierre Duhem; Henri Poincaré; fin-de-siècle physics; conventions

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Introduction

In his text “Quelques réflexions au sujet des théories physiques” (1892) Pierre Duhem wrote:

We are not alone in professing the ideas we have just set forth, and if there is an opinion which we are pleased to be able to invoke in support of ours, it is certainly that of the analyst who has written the following: “The mathematical theories are not intended to reveal to us the true nature of things; this would be an unreasonable claim. Their sole object is to coordinate the physical laws which experience teaches us, but which without the help of mathematics we could not even state”. (Duhem 1892, 165; quoting Poincaré 1889a, I)

These reflections immediately follow Poincaré’s first reflections on the status of geometries and on physical theories, published around 1890. The explanation of this agreement expressed by Duhem is one of the purposes of our article, which aims to compare the first epistemological theses of the two savants—

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philosophes. This will illuminate the history of the debate between the two (Brenner 2003, chapter III). This debate is reflected in an opposition that is explicit in The aim and structure of physical theory (1906); it manifests itself in the refusal of Maxwellian physics by Duhem and in his lack of interest in the achievements of atomism, associated with Lorentz's theory of electrons, the development of experimental microphysics (gas discharges, X-rays and radioactivity), black-body theory (Planck, Einstein), works on Brownian motion (Smoluchowski, Einstein and Perrin) and the Maxwell-Boltzmann kinetic theory; whereas Poincaré, after 1893, took an interest in atomistic theories and gave important contributions. But this dissonance must not erase the common ground of their reflections, which is related to their judgment on the state of theories around 1890.

The complex evolution of physics in this period is diffracted in the individual epistemological reflections of the savants-philosophes, who judge theories in situation. This is because Maxwell, Mach, Hertz, Boltzmann, Poincaré or Duhem try to integrate a variety of motifs: their perception of theories, their actual and personal scientific practice, a broader questioning inspired by the philosophical tradition, which imposes long lasting questions. The first reflections of Poincaré and Duhem, made before the rise of the physics of electrons and ions, have similarities which can be understood according to the contemporary state of physical theories, at a time when the Laplacian tradition, attached to a mechanistic reductionism based in the conception of the center-of-force atoms, is defeated by thermodynamics and electromagnetism, two domains to which Poincaré and Duhem are interested as researchers. The recognition of the hypothetical nature of theories, theoretical pluralism, stylistic differences between the French tradition and the physics of models, the complex nature of inter-theoretical relations and the relationship between theory and experience, constitute a common ground for questioning.  

The first part of our article concerns the changes in epistemological consensus at the turn of the century. The second part will be devoted to Poincaré's reflections on the status of physical geometries and physical theories, as they appear in his texts written around 1890. Then we analyze the first reflections of Pierre Duhem on physical theory, in particular his thesis of the hypothetical/symbolic character of physical theories and his criteria for selecting good theories, partly associated with his ideal of physical theory; the whole set of considerations, highlighting the Poincaréan inspiration. The relationships between the scientific practices of the two scholars, the contemporary theoretical context and their reflections will drive this investigation.

Changes in Scientific Consensus

Between the last years of the nineteenth century and the beginning of the 1910s, experimental access to the intimate structure of matter favored a consensus on the relevance of atomistic hypotheses of statistical mechanics. The French - notably Poincaré, Becquerel, Curie, Langevin and Perrin - contributed to this evolution, associated with the physics of electrons and ions, new radiations and Brownian motion.

In 1905, the Société Française de Physique invited H. A. Lorentz to speak on "Thermodynamics and kinetic theories". He distinguishes between two kinds of theories in mathematical physics. There are those which seek to "penetrate the intimate mechanism of phenomena" and those which, using certain general principles, establish relations between quantities "directly accessible to observation". Given the state of contemporary research, the fields of application of the two theories, which are taken as examples, are different and each one proves itself powerless where the other makes it possible to reveal relationships (Lorentz 1905, 533-534). This same pluralistic perspective can be found in Jean Perrin's book Les principes (1903), dedicated to thermodynamics. In the preface, Perrin states that molecular hypotheses correspond to the "deductive method", which "consists in imagining a priori for matter a structure whose direct perception still escapes our imperfect senses, and such that its knowledge would make it possible to deduce the

3 Poincaré was a member of the jury of the (second) doctoral thesis of Duhem (1888); the only known correspondence between the two is of this period and without epistemological interest (Poincaré 2007, 157-158). Around 1890, Poincaré saw Duhem as the great French specialist in thermodynamics (Poincaré 1892a, XIX, 233, 321-338, 366; 1892c, 63).

4 In the context of studies on Duhem, I reconstruct Poincaré's reflections and the French scientific context of the second half of the century in an unusual manner and that sheds another comparative light on their contributions; contrast with (McMullin 1990; Maiocchi 1990; Stoffel 2002).
sensible properties of the universe" (Perrin 1903, VII). Perrin emphasizes the heuristic value of this method, which makes it possible to follow "a perfectly logical march"; but he goes further:

> It seems to me that we still have the right to attribute to the molecules, atoms or corpuscles, a greater reality. And I do not fall back into metaphysics. I do not cease to forget that sensation is the only reality. This is the only reality, on the condition that all possible sensations are added to current sensations. (...) Moreover, and precisely at the moment when the interest and the legitimacy of their method were under attack, the atomists have proved it again by striking discoveries, of which the corpuscular theory has succeeded in making a harmonious whole. It seems therefore reasonable in all respects to regard the debate as settled by the reconciliation of two methods [inductive and deductive] which are by no means incompatible. (Perrin 1903, IX-X)

But this consensus, increasingly favorable to atomism and rendering energetism untenable, does not characterize the situation around 1890.

John Heilbron, in his study on fin-de-siècle physics, admitted the existence of a minimum epistemological consensus, which he calls descriptionism. This includes the common aspects of the reflections of Mach, Hertz, Poincaré, Duhem and Boltzmann, made in the late nineteenth century and inspired by Maxwell and Kirchhoff (who limited the role of physical theory to a “description” in his lectures in mechanics of 1875). Among the philosophical inspirations Heilbron refers to Kant, with his “objective idealism” which rejects access to the thing-in-itself, and the positivism of Comte. This consensus reflects the tensions between mechanics (and the associated reductionist ideal) and thermodynamics and electromagnetism, and also the success of Maxwellian physics (consecrated with Hertz’s experiments on the existence of electromagnetic waves in 1888) and the legitimization of theoretical and methodological pluralism, including the British physics of models, accepting the use of analogies and a degree of inconsistency among models and among theories, justified by fertility. From a sociological and institutional point of view, it reflects a defensive position against those who criticized science from religious and philosophical points of view and undermined its reputation in an industrial society that finances research institutions and promotes a new professional class.5

The term “descriptionism” favors a phenomenological perspective; but it hides the plurality of viewpoints on physical theories, including the persistence of atomistic beliefs. For example, in the French case, these are very present in the discussion that follows the publication in 1895, in the Revue générale des sciences, of an article by Wilhelm Ostwald, on the defeat of materialism in the sciences. The translators have called it "The defeat of contemporary atomism." It is primarily the mechanistic reductionism (matter and movement) that Ostwald criticizes, based on his vision of thermodynamics. In his reply, Alfred Cornu, Vice-President of the Académie des Sciences, renews his credo in favor of a mechanistic reductionist conception based on notions of material points and reciprocal actions. Marcel Brillouin, on his reply, advocates methodological pluralism and individualism, pointing out the success of mechanistic theories in chemistry and of the mechanical wave theory.6 As Olivier Darrigol has remarked, Heilbron’s thesis is interesting in that it tries to identify a consensus, but this consensus is not achieved around a phenomenological perspective of physical theories. It rather promotes their hypothetical nature, which is clearly present in the title of the book Science and hypothesis, which includes texts written by Poincaré until 1900 (Darrigol 2016). The epistemological reflections of Poincaré and Duhem, made during the 1890s, are also a sign of this epistemological consensus, which precedes that which will be formed at the beginning of the 20th century and which will also include atoms.7

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5 Indeed, many physicists, in their practice, abdicated sophisticated epistemological reflections and adopted a realistic spontaneous epistemology (Heilbron 1982, 56).
7 One of the central figures of this movement of recognition of the hypothetical nature of physical theories is Helmholtz (Schiemann 2009).
Poincaré's Initial Reflections: Conventions, Pluralism and Simplicity

Henri Poincaré, who built his initial reputation in mathematics, is one of the principal responsible for the revival of French mathematical physics, being recognized at the turn of the century as one of the best theoretical physicists. His courses in mathematical physics, initiated in 1885, were very successful, being translated quickly into German. The prefaces of a few of them contain important philosophical reflections to which Duhem will react, on the one hand, by showing his agreement with this mathematician who has become a paladin of the physics of principles; of another recognizing Poincaré as the central figure of the introduction in France of a British style of doing the theoretical physics which he will criticize.

Poincaré: Conventions and Geometry

The first epistemological reflections of Poincaré concern the status of metric geometries. Poincaré knows the work of Bernhard Riemann, for whom the curvature of the space in which we live must be determined by empirical measurements, those of Hermann von Helmholtz for whom the axioms are a posteriori being possible a sensible intuition of the three geometries that allow the free movement of rigid bodies, and those of Marius Sophus Lie, where continuous groups provide a rigorous mathematical elaboration of Helmholtz's researches. In 1887, in his seminal article “Sur les hypothèses fondamentales de la géométrie”, Poincaré questions the origin of Euclid's postulate of parallels and the presence of synthetic a priori judgments in mathematics. On the status of the axioms of geometry, Poincaré considers three options: they are facts of experience, or analytical judgments, or synthetic a priori judgments. He argues that none of the three options is valid and states:

Geometry is nothing else than the study of a group and, in this sense, (...) the truth of Euclid's geometry is not incompatible with that of Lobachevsky's geometry. (...) We chose from among all the possible groups a particular group to report the physical phenomena, as we choose three axes of coordinates to report a geometrical figure (...) the chosen group [the Euclidean] is only more convenient than the others and one cannot say that the Euclidean geometry is true and the geometry of Lobachevsky false. (Poincaré 1887, 215)

The general philosophical significance of this article is elaborated in an article of popularization published in 1891 in which it is shown that the ontological claims of some empiricists and the Kantian framework of the transcendental aesthetics are not acceptable. Poincaré points out that a dictionary can be constructed between terms of the geometry of Lobachevsky and terms of ordinary geometry, which makes it possible to translate the theorems of the first into theorems of the second. He summarizes his discussion by saying that geometric axioms are disguised conventions or definitions - they result from a free decision of the mind, motivated by experience (Poincaré 1891, 773). The word "convention" is used with two meanings: due to the consistency and inter-translatability of (pure) metric geometries, one can not assign preferential validity to one of them (except for simplicity reasons); the second meaning refers to the choice of a physical geometry which involves a package of coordination rules with empirical definitions. Physical geometry involves mechanics, thermodynamics (measurement standards) and optics (postulation of the rectilinear propagation of light rays). The geometry belongs to a more elementary level than the other domains (non-homogeneous or stratified holism) (Friedman 1999, 74, 80-1): the test using the parallax of the stars admits that the light rays are straight lines; but a result apparently contrary to Euclidean geometry would best be interpreted by modifying the laws of optics: "Needless to add that everyone would regard this solution as more advantageous." The choice of conventions is therefore not arbitrary, because it is based on a constraining intersubjectivity. Poincaré therefore believes that "a geometry can not be more true than another; it can only be more convenient [commode] (...) Euclidean geometry is and will remain the most convenient" (Poincaré 1891, 774).

On his sources of inspiration see: (Giedymin 1977; Poincaré 1891, 769; Heinzmann 2001; Darrigol 2007).
The Mathematical Physics Courses: 
Pluralism and the Method of Comparisons

One of Poincaré's first courses (1885-1886) is dedicated to pure kinematics and mechanisms, a traditional subject which could have helped to understand the illustrative mechanical models typical of British approaches. The course at the Sorbonne in the first semester of 1887-1888 is devoted to the mathematical theories of light, an area in which there was a clear de facto pluralism. The alternative between molecular ether and continuous ether was known since the 1830s, considering the compatibility of George Green's theory with continuous ether. Poincaré compares six competing theories and shows the equivalence of these theories despite the different physical hypotheses of departure.9

After the assertion, which Duhem will cite (see introduction above), that mathematical theories do not reveal to us the “true nature of things”, Poincaré notes that the ether is only a convenient hypothesis, neither true nor false. The several theories of mechanical ether are all “equally plausible”. To confine oneself to one of them would produce a blind confidence; the most instructive is to compare them. He notes that the molecular hypotheses, typical of French theories, “play only a secondary role. (...) I borrow from molecular hypotheses only two things: the principle of conservation of energy and the linear form of equations which is the general law of small movements (...) this explains why most of the conclusions of Fresnel remain unchanged when we adopt the electromagnetic theory of light [that of Maxwell]” (Poincaré1889a, III). In the “Conclusions”, Poincaré reinforces his instrumentalist point of view:

Besides, we cannot complain of being unable to make a choice [among the rival theories of the ether]. This impossibility shows us that mathematical theories of physical phenomena are to be regarded only as instruments of research (Poincaré 1889a, 398-399).10

Poincaré also teaches the theories of capillarity (the molecular theories of Laplace and Gauss), and the theories of elasticity, about which he distinguishes between molecular theories and phenomenological theories:

There are a great number of theories of elasticity. They can be reduced to two classes: in the first class we will class theories based on molecular hypotheses; in the second, those whose authors have sought to free themselves from all hypotheses on the intimate constitution of bodies; these latter theories are generally based on thermodynamics. (Poincaré 1892c, 27)

Poincaré points out that the two methods lead to the same equations, but that the molecular hypotheses are speculative (Poincaré 1892c, 62, 64). In the preface to his *Thermodynamics* he seems to be more explicit about the demise of molecular explanations:

Abandoning the ambitious theories of forty years ago, encumbered by molecular hypotheses, today we are seeking to build upon thermodynamics alone the whole edifice of mathematical physics. (Poincaré 1892a, V)

The course of the second semester of 1888-1889 is dedicated to theories of electrodynamics and to Maxwell's electromagnetic theory of light; after that, Poincaré will deal with the theory of Helmholtz and the experiments of Hertz. He will continue with his method of comparisons, showing, for example, that Maxwell's theory is a special case of that of Helmholtz (Darrigol 1993, 215, 222; 1995, 5-8); he notes that comparisons (mathematical ones) must not make us forget the distinct physical senses attached to the theories:

Hertz considers that the very substance of Maxwell's ideas lies in the equations he obtains, and that a theory may be regarded as equivalent to that of Maxwell, provided that it leads to the same

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9 On the history of ether theories see Schaffner (1972).
10 The underdetermination of the theories of the ether is here the result of the linear character of the equations (Poincaré 1889a, 398-400); this will favor Poincaré's skepticism towards ether, and his preference for the views of Hertz, who abolished the ether (Darrigol 2000, 356).
equations. Thus Helmholtz's theory contains, as a special case, that of Maxwell, and yet Maxwell would not have accepted this interpretation, in which actions at distance still play a part. (Poincaré 1892b, VI)

In spite of his admiration for Helmholtz, Poincaré will recognize with Hertz that Helmholtz's theory is contrary to the unity of the electric force; this unitary conception based on considerations of symmetry (and not of experimental origin) justifies Poincaré's preference for Maxwell (Abrantes 1985, 208-212; Darrigol 2000, 355). From the 1890s, Helmholtz's theory will no longer be considered as an alternative to Maxwell's, despite Duhem's efforts (Atten 1992, 10-11 and 444-448).

**Maxwell's *Treatise*: Its Fundamental Idea, National Styles**

In the preface to his lectures on the theories of Maxwell (1889), Poincaré wants to show that the *Treatise* contains a fundamental idea, despite the fact that "the English scholar does not seek to construct a single, definitive and well-ordered edifice"; in fact "it seems rather that it raises a large number of temporary and independent constructions, between which communications are difficult and sometimes impossible" (Poincaré 1890, VIII). He notes the advantages of this method, which he attributes to Maxwell:

> We must not, therefore, flatter ourselves with avoiding all contradiction; (...) indeed, two contradictory theories may, provided they are not mixed up, and that they do not seek the substance of things, may be both useful instruments of research, and perhaps reading Maxwell's *Treatise* would be less suggestive if it had not opened to us so many new divergent paths. (Poincaré 1890, IX)

According to Poincaré, this appearance of fragmentation conceals the fundamental idea of the *Treatise*. It corresponds to a profound change in the concept of mechanistic reduction of phenomena:

> Maxwell does not give a mechanical explanation of electricity and magnetism; he merely demonstrates that this explanation is possible. (...) If a phenomenon includes a complete mechanical explanation, it will include an infinity of others which will also give a good account of all the peculiarities revealed by experience. (Poincaré 1890, VII, XIV)

This theorem, stated in Maxwell's *Treatise* (Maxwell 1873, II, §831), is demonstrated in detail by Poincaré, using the Lagrangian formalism. The generalized coordinates correspond to observable/measurable parameters. These can be related to an unobservable molecular coordinate system. There are an infinity of such systems which, by transformation of variables, allow us to obtain the same Hamilton function, \( T + U \). This theorem justifies the attitude of those who find the "complete explanations" (typical of the ideal of the Laplacian program) unnecessary, especially the speculations on the ultimate structure of the ether. But the opposite attitude remains valid:

> Among all these possible explanations, how can we make a choice for which the help of experience is lacking? Perhaps a day will come when physicists will lose interest in these questions, inaccessible to positive methods, and abandon them to the metaphysicians. This day has not come; man does not resign himself so easily to ignore eternally the substance of things. (Poincaré 1890, XV)

Then Poincaré remarks how ontological preoccupations about the nature of the substance or the inner mechanism are alien to the spirit of Maxwell's fundamental idea:

> The same spirit is found throughout the work. What is essential, that is, what must remain common to all theories, is brought to light; anything that would fit a particular theory is almost always ignored. The reader thus finds himself in the presence of an form almost empty of matter which he is at first tempted to take for a fleeting and elusive shadow. But the efforts to which he is thus condemned

compel him to think, and he ends by understanding what was often a little artificial in the theoretical
ensembles he once admired. (Poincaré 1890, XVI)

The epistemological value of the theorem is emphasized by Poincaré, who sees in it an explanation of the
indecision between rival theories present in historical cases:

The preceding is confirmed by the history of all the parts of physics; in optics, for example, Fresnel
believes the vibration perpendicular to the plane of polarization; Neumann regards it as parallel to
this plan. We have long sought an "experimentum crucis", which enabled us to decide between these
two theories, and we could not find it. All these facts are easily explained by the properties of the
Lagrange equations which I have just recalled. (Poincaré 1890, XIV)

The explanations that postulate unobservable quantities are therefore a cause of underdetermination of
theories. Also, the theorem highlights the abstract dynamics, favored by some of the British physicists.
Among the old theoretical ensembles we find the molecular physics of Laplace. Poincaré establishes
a close relationship between the tradition of French mathematical physics and the difficulties in
understanding Maxwell's works:

The first time that a French reader opens Maxwell's book, a feeling of uneasiness, and often even
mistrust, mixes at first with his admiration. (…) Why do the ideas of the English scholar have so much
difficulty in acclimatizing among us? It is no doubt that the education received by most enlightened
Frenchmen disposes them to taste precision and logic before any other quality. The ancient theories
of mathematical physics gave us a complete satisfaction in this respect. All our masters, from Laplace
to Cauchy, proceeded in the same manner. Starting from clearly stated hypotheses, they deduced
all the consequences with mathematical rigor, and then compared them with experience. (Poincaré
1890, V-VI)

Poincaré emphasizes here the style of presentation, the logic and precision favored by the training of French
physicists. He also recognizes the persistence of the Laplacian program, which contains an ontology of
center-of-force atoms, the conception of an unobservable matter, "which have only purely geometric
qualities and whose atoms are nothing but mathematical points subject to the laws of dynamics" (Poincaré,
1890, VI).

Simplicity and Coordination of the Rapports Vrais

Poincaré seems to favor a physics in which the "differential equations deduced from experience can be put
into the Lagrangian form" (Poincaré 1890, XII); and he questions the merit of the complete mechanical
explanations, since as soon as one is sure of having the Lagrangian version, Maxwell's theorem guarantees
the existence of a myriad of such explanations, which takes away their value. But it is more complicated
than that. Poincaré indeed values the unifying role of atomist hypotheses – for example, in 1892 he points
out that Helmholtz's (1858) theorems on a perfect liquid, implying that the vortex rings "must retain their
individuality," inspired William Thomson (1867) to conceive an atomic theory of matter based on a universal
perfect liquid, which would allow "a mechanical explanation of the universe" (Poincaré 1893c, 2). In addition,
while Poincaré valorizes the Lagrangian approach, he questions his universality by showing that the second
principle remains rebel even to this more phenomenological mechanistic approach.13 Indeed, from 1893,
Poincaré will favor the statistical approach of Maxwell and Boltzmann (Príncipe 2008, 293-334). His
conception of mathematical physics is rather flexible, favoring the critical and comparative appreciation
which presupposes methodological pluralism and a kind of suspension of judgment.

12 Poincaré alludes to the experiences of Wiener (Poincaré 1891b; Langevin in Collectif 1914, 73).
13 Admitting that Thermodynamics can be presented according to the Hamiltonian formalism, Poincaré believes to have
demonstrated that no function of the state of a system governed by Hamilton's equations can be constantly increasing
– the Clausius principle is incompatible with that of the Least Action (Poincaré 1899b; Príncipe 2008, 279-293; Príncipe
2014, 135-137).
However, Poincaré’s pluralism, like that of Maxwell, is not a variety of relativism that would fragment the body of physical theory because, beyond the differences of individual and national style, Poincaré postulates an intersubjectivity based on constitutive principles (mathematics being a constitutive language from this point of view) and on principles of convenience (in the sense of Kant) that allows us to determine which theories are the simplest and most harmonious. For example he remarks:

Our choice can therefore be guided only by considerations in which the share of personal appreciation is very high; there are however solutions that everyone will reject because of their quirkiness and others that everyone will prefer because of their simplicity. (Poincaré 1890, XII)

In the preface to his *Thermodynamics*, Poincaré emphasizes the role that considerations of simplicity have in the construction of theories. Poincaré affirms that our mind is endowed with a faculty which is the condition of possibility of science: the faculty of generalizing the empirical data. It allows us to satisfy our need for order and harmony and at the same time allows us to foresee. The laws are formulated "after relatively few experiments and which present certain divergences". Since "every proposition can be generalized in an infinite number of ways", the choice of the general law is made according to our criterion of simplicity, by obeying "a necessity to which the human spirit cannot escape" (Poincaré 1892a, VI, VII). It is simplicity that favors the acceptance of the principles of thermodynamics:

The imposing simplicity of the principle of Mayer [energy conservation] also contributes to affirm our faith in it. In a law deduced immediately from experience, such as that of Mariotte, this simplicity would appear to us rather a reason of mistrust, but here it is no longer the same, for we see elements, disparate at first glance, to be arranged in an unexpected order and to form a harmonious whole. (Poincaré 1892a, VIII)

Poincaré also remarks the advantage of teaching the historical course of "long groping by which man arrives at the truth (...) We shall note the important role played by various theoretical or even metaphysical ideas" (Poincaré 1892a, V).

Poincaré believes that the synchronic and diachronic comparison of theories allows the sedimentation of laws, which he will later call true relations [*rapports vrais*] (Poincaré 1900, 1168; 1902a, 292-3). Theories coordinate physical laws; for example, "the laws of optics and the equations which translate them analytically (...) will remain true, at least as a first approximation" (Poincaré 1889a, I-II), (Poincaré 1892b, VI). In his *Thermodynamics* he says: "The accuracy of physical laws is always limited by observation errors. But at least they pretend to be first approximations and we hope to replace them gradually by more and more precise laws" (Poincaré 1892a, XIII). Poincaré understands the complex nature of theories, which derives in large part from the multiplicity of inter-theoretical interrelationships, but he believes in the human capacity and need to reduce complexity in an unforeseen historical march towards the systematic unity of theories, which makes it possible to us to identify structures of true relations, subjects that he will deepen in his later texts, especially in those that will form *Science and hypothesis*.

### Duhem: Physical Theories Faced with the Bankruptcy of Mechanics

In 1892, Duhem published his first major epistemological reflection, his opening lecture of the *Course of mathematical physics and crystallography at the faculty of sciences of Lille*, and also the first part of his *Commentary on the principles of thermodynamics*, with which he continued his research program whose aim was the creation of a "general theory of material transformations, which encompassed physical sciences" (Bordoni 2012, 11). The agreement with Poincaré is explained in the *Commentary*:

Every physical theory rests on a certain number of definitions and assumptions, which are, to some extent, arbitrary; it is therefore permissible to attempt to expound such a theory in a logical order; but to claim that he was given the only logical order of which it is susceptible would be an unjustifiable claim. (...) We are convinced that the principles of thermodynamics can be chained in a way other than that which we have adopted and yet also satisfactory, perhaps more satisfactory. (...) If the
question we have examined seems rather philosophical, let us be permitted to invoke (...) the interest shown (...) by an illustrious analyst [Duhem means (Poincaré 1892a)], for researches which concern the principles of thermodynamics. (Duhem 1892c, 270)

In this work, Duhem explicates and clarifies the basic hypotheses of thermodynamics, starting from a strictly phenomenological conception, notably by presenting an "axiomatic treatment of the first law of thermodynamics which is surprisingly good by present day standards" (Miller 1970, 229). Duhem calls "conventions" the introductory axioms, which will make it possible to obtain a mathematical expression symbolizing the transformation of a system in the presence of foreign bodies. He notes that the nature of this contribution of the action of foreign bodies to the energy of the system remains obscure. To penetrate its nature "is not the object of physics but of metaphysics" (Duhem 1892c, 290). The word "convention" appears here in the framework of a theory with principles, being associated with "axioms" and with the criticism of the hypotheses of a theory, themes present in Poincaré's reflections.14

Definitions and Hypotheses

Let us follow the considerations of Duhem's first epistemological text (Duhem 1892a). Duhem considers the classical Amperean idea that raw facts are organized by experimentation, which is the beginning of an ascending classificatory march that leads from facts to laws and from laws to theories (Braverman 2016, 71-72). A theory is constituted by a series of operations, the first being the definition of the quantities which symbolize the corresponding physical notions: for example, temperature symbolizes the notion of heat. The choice of the physical quantity is "to a high degree arbitrary", because "between these two ideas, being warm and temperature, there is no kind of natural relationship (...) Physical definitions constitute a true vocabulary (...) Definitions are a set of conventions matching a magnitude to each physical notion" (Duhem 1892a, 143-144). In the Commentary, Duhem shows how the construction of a physical notion, although starting from sensory experience, mobilizes the abstraction that corrects the logical imperfections that stem from the limited nature of the sensations associated with our organs:

This property of the bodies which we characterize by the words: to be hot, to be cold, to be more or less warm, our faculty of abstraction is soon going to attribute to it characters that sensation does not give us. (Duhem 1892c, 284)

The concept of thermal equilibrium, essential for the construction of the concept of temperature, presupposes the concept of an isolated system, which is an abstraction (Duhem 1892c, 274, 285), and results from generalization from vulgar observations. Once the "law of thermal equilibrium" is established (for an isolated system to be in equilibrium all its material parts must be equally warm) it "leads us to correct the data of our sensations (...) our sensations do not always inform us of the degree of heat of a body" (Duhem 1892c, 285-286). This is why Duhem emphasizes that experimental physics rises above empiricism (Duhem 1892a, 140).

Duhem considers that the correspondence which must be constructed between the notion of "being warm" and "temperature" can only be a partial analogy. It takes up the distinction between quantity and quality (Aristotelian distinction which is related to the Kantian distinction between extensive and intensive quantities); this distinction is present in the considerations of Maxwell, Mach and Helmholtz concerning the discussion of the concept of measurability, and in particular of that of temperature. Helmholtz (1887) believes that "intensive quantities, for which no concrete addition is known, could only be measured through a connection with extensive quantities" (Darrigol 2003, 519). Duhem judges that the establishment of this necessary connection introduces an arbitrary element. The "being warm" property is not a quantity because

14 In his justification of the phenomenological approach, he invokes a criterion of simplicity, in a passage of Poincarean flavor: "In Physics, it is both impossible and useless for us to know the real constitution of matter. We are simply trying to conceive an abstract system that provides us with an image of the properties of bodies. To construct this system, we are free to represent a body which seems to us continuous either by a continuous distribution of matter in a certain space or by a discontinuous set of very small atoms. The first mode of representation, leading in all parts of physics to simpler, clearer, and more elegant theories, will be preferred to the second." (Duhem 1892c, 272)
it is not susceptible to addition; the correspondence between this property and an algebraic quantity is arbitrary because numbers have properties that do not correctly represent the properties of the corresponding physical notions: "We do not understand what it means (...) body A is seventeen times warmer than body B" (Duhem 1892a, 142). The characters that are required by the correspondence (to respect the zero law of thermodynamics and the transitivity of the relation "body A is warmer than body B") leave the temperature defined modulo a continuous and strictly increasing function. The material concretization of the correspondence depends on postulates defining what is a thermometer, admitting that a quantitative property of the thermometer depends only on temperature. Maxwell, Mach, Poincaré and Duhem thought that thermometers should be considered "as purely conventional means to identify and order thermal states" and, in addition, Mach and Duhem (...) believed that the confusion between quality and quantity belonged to the mechanical reductionism they both condemned" (Darrigol 2003, 519).

After the first operation (definition of the quantities that symbolize the physical notions), a second operation is then implemented: hypotheses relate the physical quantities and by mathematical deduction, we obtain consequences that are tested experimentally (Duhem 1892a, 145).

Duhem does not believe possible the existence of theories without hypotheses and criticizes the "hypotheses non-fingo" of Newton and Ampère, the idea that theories can be deduced from experience alone:

What then has Newton done to formulate the law of universal gravitation? (...) He took as a hypothesis a proposition of which the experimental laws [the laws of Kepler] placed at the beginning of his theory are only particular consequences, exact or simply approximated. This is the general method employed by all theorists. To formulate their hypotheses, they make a choice of some of the experimental laws, the whole of which must be embraced by their theory; then by means of correction, generalization, and analogy, they compose a proposition of which the laws are exact or simply approximated consequences, and it is this proposition which they assume. (Duhem 1892a, 148)

The paths of theoretical invention are multiple and as soon as an hypothesis makes it possible to deduce a wide range of consequences, it is not necessary that it directly symbolizes the experience, although it is its relation to experience which gives it physical meaning. As in the case of the definition of quantities, there is arbitrariness in the choice of hypotheses. Duhem will be particularly concerned with the question of the criteria for choosing hypotheses, combating arbitrariness.

A first case of arbitrariness is the one resulting from the conventional choice of the definitions that allow the measurement:

In order to represent the same notion, one can in general make use of a multitude of extremely different magnitudes (...) the simple change of the definitions would already lead to changing the hypotheses [which would correspond to translate] the same hypothesis by means of different symbols, and these two statements of the same hypothesis in two different systems of symbols do not constitute any more two different hypotheses than the statements of the same proposition in French, Latin and Greek constitute three different propositions. (Duhem 1892a, 152)

This adjustment between definitions and hypotheses reminds one of Poincaré’s reflections: Poincaré speaks of the intertranslability of metric geometries and the fact that the choice of a physical geometry (which makes it possible to measure lengths and angles) implies a set of rules of coordination with empirical definitions; Duhem notes that the conventional choice of definitions (which allow measurement) implies adjustments in the hypotheses, but that these adjustments are related to each other as translations in different languages of the same idea. Also their discussions on the concept of temperature (their common sources being

15 On the conventional character of the equality of temperatures in (Poincaré 1892a) see Darrigol (2003, 563). Darrigol shows that Helmholtz inspired the conceptions of Poincaré and Duhem on measurement; Duhem is individualized by his ideal of a physics of qualities, a neo-Aristotelian conception according to which qualities remain irreducible to quantity – "a property identified as a quality had to remain a quality for ever" (Darrigol 2003, 568), thematic preference (in the sense of Holton) to which Duhem remains faithful throughout his career. On the history of the concept of temperature see also (Chang 2004).
Maxwell, Mach and Helmholtz) exhibit the need for conventions in order to measure.\(^{16}\)

### The Good Theories

In a good theory, the consequences of the axioms form a complete and varied set, which demonstrates the ability of the theory to coordinate/symbolize experimental laws (Duhem 1892a, 145-146).

Duhem acknowledges the existence of different sources of indeterminacy of theories (Duhem 1892a, 149-151). Firstly, the hypotheses of a theory go beyond the simple symbolic translation of experimental laws, introducing modifications that are the work of the scientist's mind. Secondly, the presence of experimental error allows for competing theories, that yield different laws and yet agreed with experience, in the interval of experimental uncertainty. Thirdly, the extension of a theory may not be well known: a theory is designed to be applied in a certain domain, and its extension to a larger domain may not be appropriate. Fourthly, a theory is useful or good depending on the precision required in its applications – the simple gas law of Gay-Lussac, the generality of which has been invalidated by the experiments of Victor Regnault, may remain good for a chemist or an engineer.

Among these considerations, Duhem notes the role of experience in accepting or rejecting a theory. If there is a discrepancy between the expected consequence of the theory and "the methods of observation of which the theory accepts the control, the theory must be condemned" (Duhem 1892a, 151).\(^{17}\)

The sources of indeterminacy allow us to understand the theoretical changes studied by the history of science. The value of a theory being relative (or conditional), a theory might be good and yet be replaced at the same time by a better one, either because the last one is capable of representing a wider class of laws, or because it is capable of represent the same laws with a greater degree of approximation. This substitution can be obtained either by a more continuous process, which maintains the hypotheses of the first theory by adding new parameters or some new hypotheses; or by a process which requires deeper modifications "which alter the definitions and assumptions upon which the first theory was based" (Duhem 1892a, 152).

According to the preceding considerations, the value of a theory depends on the examination of the extent of the domain of a theory, of experimental uncertainty, and of the concrete use of a theory (its more immediate instrumental character). Duhem proposes other more internal/logical criteria to make the choice between competing theories:

Logic leaves the choice of hypotheses free; but it requires that all these hypotheses be compatible among themselves, that they are all independent of one another; a theory has no right to invoke unnecessary assumptions; it must reduce its number to a minimum; it has no right to bring together consequences deduced from irreconcilable assumptions. (Duhem 1892a, 166; see also 169)

Until now, Duhem, in his overview of the criteria for evaluating the value of a theory, remains in the consensual plan of a certain good sense, as long as we accepts the central role of hypotheses and of the convenient definitions needed for measurement. This plan is abandoned when Duhem judges contemporary theories. Firstly, after citing the passage of Poincaré on the French style of presentation of the theories (see above), Duhem sees a weakness of the mind in the style of the Treatise of Maxwell (Duhem 1892a, 168), which implies a mixture of irreconcilable hypotheses -- a subject developed in a subsequent text on the English School (Duhem 1893); in the latter text, among his long considerations, Duhem does not tell us his opinion on what Poincaré considered the fundamental idea of Maxwell's Treatise, most probably because it is in harmony with the research program of Duhem: the Lagrangian approach valorizes a phenomenalist

\(^{16}\) There is a passage in which Duhem alludes to Helmholtz's considerations on non-Euclidean geometries (Poincaré refers them to the end of his article of 1891) and on arithmetic and the problem of measurement: "these profound researches on foundations of geometry and these meditations, so satisfying to the mind, concerning the origin of the axioms of arithmetic" [reference to Zahlen und messen, published in 1887] (Duhem 1893, 375).

\(^{17}\) I am the one who emphasizes. Experimental control is not therefore a simple and immediate action. In the Commentary, considering "absolute movement", Duhem points out that supplementary hypotheses are always associated with the assumption that a trihedron has its axes absolutely fixed (Duhem 183c, 271), which prefigures the broader discussion of (Duhem 1894) and makes one think of Lakatos' protection belt; see Leite (2017, 145).
view in which mechanics has a more abstract structuring role. For Duhem there exists an ideal form of theory: the hypotheses of a theory must be the symbolic translation of experimental laws, in which case the theory is modified by a continuous process. When hypotheses move away from experimental laws, theories become more vulnerable to demolition (Duhem 1892a, 153). Implicitly, he sees an opposition between phenomenological theories and theories that postulate unobservables, which is typical of atomic mechanical theories. Duhem believes that this last class of theories has fulfilled its historical function and is in a state of rupture.

The Bankruptcy of Mechanical/Atomic Theories

Around 1870, in chemistry, atomic hypotheses were rejected by the adepts of the theory of equivalents. The atomist Adolphe Wurtz was opposed to Marcellin Berthelot and to Henri Saint-Claire Deville. In this connection, two debates took place at the Académie des sciences (1876, 1877) between physicists and chemists; the first one had as a pretext the result of Kundt and Warburg’s experiments on the specific heat of mercury vapor (the ratio between the specific heats obtained, \( \gamma = 5/3 = 1.66 \), corresponding to a monoatomic gas according to the predictions of the kinetic theory), the second concerned the law-hypothesis of Avogadro and the law of Dulong and Petit for specific heats (base of the atomic hypothesis in chemistry). The debate made it clear that the elite of French physicists was predominantly in favor of atomic hypotheses (Príncipe 2008, 190-200).

In 1892, in his article on atomistic hypotheses, Duhem took the side of the equivalentists, judging that atomistic hypotheses produced only “difficulties which had arisen from the presumptuous desire to take a classification for an explanation.” Duhem cites in his favor the ideas of Sainte-Claire Deville (1818-1881) for whom chemistry must follow a method not "in the manner of geometrical concepts, but in the manner of naturalists", that is, a method of classification (Duhem 1892b, 452). In his Lessons on affinity of 1867, the mentor of Jules Moutier speaks of atomistic hypotheses as “contemporary tendencies to abstraction” to which we ought not to give reality:

Let us gradually do a work of classification that will be for a long time, that will perhaps always be, incomplete (...) But we must never rely on hypotheses that last only a moment (...) All the hypotheses accepted today will necessarily disappear from science. I make no exception, even for (...) the hypothesis of the luminous ether. (Sainte-Claire Deville cited in Duhem 1892b, 453)

This agreement means that Duhem has a unitary vision of chemistry and physics, inspired by a phenomenological methodology and translated by his notion of natural classification.18

Duhem recognizes that the ideal of mechanistic reductionism is a majority trend in France (Duhem 1892a, 153, 154; Príncipe 2015c). Here is a definition of mechanical theories:

To each physical notion, the theory had to substitute, as a symbol, a certain magnitude. This magnitude needs to present certain properties, the immediate translation of the characters of the notion which it symbolizes; but, apart from these characters, which in general are few in number, its definition remains absolutely arbitrary. In a mechanical theory, one imposes in addition to all physical magnitudes (...) the condition of being composed by means of geometrical and mechanical elements of a certain fictitious system; to all hypotheses, to be the statement of the dynamic properties of this system. (Duhem 1892a, 154)

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18 Jules Moutier, Duhem’s professor at Collège Stanislas (Paris), was a disciple of Deville. Duhem found in Deville the French pioneer of physico-chemistry: to build a chemical mechanics, based on thermodynamics, “it sufficed that Berthollet’s main idea was revived and that it was irrefutably established that the laws governing physical changes and the laws governing chemical reactions are of the same nature. That was achieved by the work of Henri Sainte-Claire Deville” (Duhem 2002 [1899], 264). This text continues attributing to the “High priest of official science” – allusion to Berthelot – the responsibility for the difficulties in the development of physical chemistry in France; see also (Klein 1990, 53-54).
Duhem considers that a serious disadvantage of these theories is "the obligation to include in these definitions and hypotheses only a very limited number of notions of a determinate nature", and there is no guarantee that "all experimental laws can be symbolized by a combination, even very complicated, of mechanical concepts alone" (Duhem 1892a, 156, 157).

This question is natural in the theoretical context of that time. Duhem illustrates his judgment on the impasse to which the ideal of mechanical theory leads by three contemporary examples: the theories of the ether, those of heat and those of electricity. The first example is that invoked by Sainte-Claire Deville and which has aroused the reflections of Poincaré in the preface to his lectures on the theories of light. The luminous ether has been constructed using the theories of elasticity, the ether being conceived by some as a continuous medium and by others as being formed of isolated atoms (Duhem 1892a, 155). Considering the mechanical theories of heat, Duhem alludes to the mechanical analogies formulated by Clausius (1871) and studied by Moutier around 1875 (Príncipe 2008, chapter 6), and those of Helmholtz (1884 and 1886) criticized in (Poincaré 1889). Duhem concludes, paraphrasing Poincaré, that they cannot "give a satisfactory account of the principle of Carnot" (Duhem 1892a, 157). According to Duhem, Maxwell, in treating the subject of electricity, formulates several contradictory theories, imagining mechanical media with very complicated properties, and that are incompatible with the well-established theories of hydrostatics and elasticity (Duhem 1892a, 156, 168).

Duhem expresses his conviction that "mechanical theories disappear from science one after the other" and this is due to the fact that "among the hypotheses upon which a mechanical theory rests, there are a great many which have no source in experience and which arise only from the demanding conventions arbitrarily laid down by the physicist" (Duhem 1892a, 157). He therefore suggests that, contrary to the case of those conventions that must be introduced with the definition of physical magnitudes, one must eliminate the conventions attached to hypotheses and which are only a consequence of the work of the mind of the physicist.

Conclusions

The object of the article is to show a certain proximity of Duhem to Poincaré in his first philosophical reflections.

The agreement with Poincaré concerns global aspects of physical theories. The theory represents an economy for the mind: "The theoretical science aims at relieving the memory and helping it to retain more easily the multitude of experimental laws" (Duhem 1892a, 140); it is intended to provide a systematic classification or representation of experimental laws, not pretending to provide "a metaphysical explanation of the material world" that would "contemplate the very structure of the world" (Duhem 1892a, 150, 158-159). However, like Poincaré, Duhem recognizes the importance of the interactions between metaphysics and physics (and their criticism), a recurring subject of his historical work (Leite 2013).

Theories are constructed from hypotheses, taken as the starting point of the mathematical deduction, and they do not have to be a mere translation of experimental laws. Like Poincaré, Duhem distinguishes between phenomenological theories and (mechanical) theories that postulate unobservable entities. Both value mathematical physics and its tool: mathematics. Mathematical analysis, Duhem writes, is "a necessary instrument for the construction of any physical theory (...) and the physicist must be able to use, if necessary, all the parts of this instrument". At the same time mathematical analysis deserves to be cultivated by itself because of its beauty, and because sometimes its internal improvements end up rendering service in other fields of physics (Duhem 1892a, 171-173).

The points of convergence between Duhem and Poincaré are quite numerous, since more than half of the "Quelques réflexions (...)" illustrate this agreement (Duhem 1892a, 139-165). Duhem differs from Poincaré only in his appreciation of the various methodologies or styles of making theory (Duhem 1893; see Maiocchi 1992, 377-380); and he makes judgments that reflect his ideal of a more phenomenological theory

19 This is another resonance; Poincaré will say: "Mathematics have a triple goal. They must provide an instrument for the study of Nature. But that is not all, they have a philosophical aim, and, I dare say, an aesthetic aim" (Poincaré 1897, 857).
and his constant concern for global coherence and uniqueness of representation.\textsuperscript{20} Although the comparison given in this article is limited to the early 1890s, let us point out some factors of the progressive separation of Duhem and Poincaré.\textsuperscript{21} In the following years, Poincaré frequented the neo-Kantian and republican milieu, becoming one of the collaborators of the \textit{Revue de métaphysique et de morale} (a journal in which Duhem published only once, in 1916), see (Souléi 2009, 68, 222-225), (Príncipe 2015a et 2015b). It seems to me probable that Duhem’s divergence from Poincaré’s ideas, explicit in his later texts, is partly an effect of his rejection of neo-Kantianism, although Poincaré’s interest in the progress of atomism may have played a more important role. Duhem frequented neo-Thomist circles hostile to Kantianism, see (Rossi 2006, 123, note 35). Poincaré’s flexible epistemological views are in harmony with his anti-dogmatism, his opposition to clerical intolerance, his republican spirit in favor of the egalitarian ideal, free thought and the right to seek and to speak the truth.\textsuperscript{22}

References


\textsuperscript{20} Duhem expressed his initial admiration (see my introduction) for the philosophical reflections of Poincaré elsewhere; for example, in his intervention during the third International Catholic Scientific Congress in Brussels (September, 1894), where he was attacked because of his reflections on the relations between physics and metaphysics; Duhem considers that “metaphysicians should have a knowledge of physical theories, acquired by ten to fifteen years of first-hand experience, rather than by reading prefaces to physics textbooks, before seeking to define the relation of physics to metaphysics: "If you want to make the philosophy of sciences, be a Helmholtz or a Poincaré!” ["Si vous voulez faire la philosophie des sciences, soyez un Helmholtz ou un Poincaré!”] (Hielbert 2000, 318). Simultaneously, the two criticize each other's contributions to thermodynamics, see note 1 above and (Stoffel 2002, 344, note 54).

\textsuperscript{21} The word “convention” does not appear in \textit{The aim and structure of physical theory}; It is also the case of the passage (Duhem 1892a, 152), cf. above end of 3.1, which brings him closer to Poincaré. Duhem wanted to eliminate all that could bring him closer to the geometrical conventionalism of Poincaré interpreted as a pragmatist vision.

\textsuperscript{22} See: (Mawhin, 2004, 11-13), (Toulouse 1910, 143-144). In the Dreyfus affair they took opposing parties (Stoffel 2002, 46).


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